The Quaternion

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Leopard Spots

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almost all reals must undefinable.

One can see the other side of the coin in a typical undergraduate course in linear algebra. The proofs and concepts are built as concretely and predictably as the bricks in Wrigley Field's wall. I cannot imagine an emergent truth being found here.

On the other hand ... In his beautiful little book "A Primer of Real Functions," Ralph Boas used the Baire Category Theorem to prove that almost every continuous real function is nowhere di□erentiable. And, L. E. J. Bouwer proved that every continuous function f mapping a the unit interval $[0, 1]$ into itself has a fixed-point (i.e., a point x such that $f(x) = x$). But even the statement given here is unexpected when first encountered. These two results fit my notion of emergent.

Of course, I've still only hinted at the idea of emergent truth in mathematics. The term "emergent," as I've used it, originated in biology where it means *unexpected, or beyond, reductionist explanation*. Theoretical biology may have a wealth of biochemistry and genetics, but life, what ever that is, remains an emergent phenomena both biologically and mathematically. It is not yet reducible to molecular mechanics. Recent growth in dynamical systems, and the easy computer simulation of complex systems, has lead mathematicians to theoretical biology and biological information processing. These parts of biology are especially rich in ghostly and unexpected phenomena. A search of the world-wide-web using the keywords: *emergent*, *mathematics*, and *biological information processing* returns thousands of hits.

While emergent phenomena are *seen* at the global level, emergent phenomena *arise* from local structure, with no well defined intermediate structure; this is the source of the mystery.

Half a century ago, John von Neumann and Arthur Burks investigated a major biological phenomenon: self-reproduction. Von Neumann's goal was to develop a mathematical insight into the nature of self-reproduction. The formalism he developed evolved into what is we now call cellular automata: networks of small, identical processors carrying out some kind of global computation.

From cellullar automata, we reach possibly the most famous emergent phenomenon: "how does the leopard get its spots?" Allan Turing stated and solved this problem. He wanted to explain how a network of embryonic leopard skin cells can develop a pattern by themselves. This tissue consists of billions of cells, each with the computational complexity of an early personal computer. They exchange information with neighbors, so there is a computer network of sorts. But, being biological in nature, members of this network do not change state in lock-step: the actions of individual cells are not synchronized to those of their neighbors. And there is no regular pattern of communication: one cell might have 16 neighbors while another has 24. Finally, "random cell-activity" means that the computation is non-deterministic!

At the level of cells, or thousands of cells, Turing's network of cells could be said to be amorphous. Yet a color-pattern of similar spots emerges in the fur growing out of the skin. The spots of this pattern contain of tens of millions of cells. And biological evidence indicates that the pattern information telling each cell its place in the pattern does not exist until fairly late in embryonic development. So where does the pattern come from? How does the network compute it?

One might think that there should be a proof that the pattern cannot exist. One would expect that irregularities in coloring should exist on such a small scale that the pelt should be uniformly muddy. Yet using mathematical analysis in a proof similarly to the one that most real numbers are undefinable, one finds the small scale irregularities almost always generating large scale regularities. *While emergent phenomena are* seen *at the global level, emergent phenomena* arise *from local structure, with no well defined intermediate structure; this is the ghostly mathematics*.

Turing first solved the problem in 1952 in a seminal paper now cited as a cornerstone in the development of theoretical biology. It used systems of reaction-di□usion di□erential equations to explore the mechanisms of morphogenesis. And four decades later, Turing's mechanism was shown 7(A)28(t)-055e469(b)

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Leopard Spots

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natural phenomena. Such "asynchronous" activity must be controlled by cell programs which reduce neighborhood entropy (disorder). Entropy-reducing asynchronous networks can do things that synchronized networks cannot! This is certainly a surprising.

When life is viewed as a form of computation and its power is estimated mathematically, it surpass our most powerful supercomputers. In the awesome book "Infinite in All Directions," Freeman Dyson calculates that a 10^{23} bit computation is required for the simplest act by a human (together with the mechanisms of life while performing the act). And a reasonable estimate, by this author in "Computing With Biological Metaphors," of the computational power of our skin is about 1 % or 2 % of that of our brains!

The conceptual distance between physical foundations and these phenomena makes them emergent.

People

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Mourad Ismail has been travelling a lot, including Hong Kong, where he organized a "Summer School in Applied Analysis." He was also appointed an editor of the Journal of Physics A.

Manoug Manougian wrote an award winning documentary "The Genocide Factor: The Human

Let us hear from you

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(and future USA President) Dwight Eisenhower that the faculty *are* the university. This is only half true: the students, past, present, and future, are also the university.

We the faculty would like to hear from former students. We maintain a website at

`http://www.math.usf.edu/`

but you can also contact us at

`mathdept@math.usf.edu`

or by phone (813-974-2643) or fax (813-974-2700) or by US mail (Publicity, Department of Mathematics, 4202 E. Fowler Ave. PHY114, Tampa, FL 33620).

A little help from our friends

We are trying to develop a first class teaching and research program with very limited resources. While we are working on regular (official) funding, any little bit we can get helps. If you would like to support the Department, please contact Denise Marks at (813) 974-2643; we can take checks made to the *USF Department of Mathematics*, and if you can designate the funds to go to some place specific (e.g., the Center for Mathematical Services, colloquia, the Nagle Lecture Fund, scholarships, etc.) if you want.