The Quaternion

The Newsletter of the Department of Mathematics, USF–Tampa

Volume 14: Number 1 Fall, 1999

Feature Article: Approximations at USF

by Vilmos Totik

V. I Arnold, one of the leading mathematicians of our time, flatly defines mathematics as part of physics. According to this anything in mathematics that is irrelevant to the outside world is useless and avoidable.

But mathematics has its own ways of expanding. Internal coherence, easthetics, and the desire for completeness, coupled with curiosity and ingenuity of the researchers are the main driving forces behind new concepts and theories. It is a miracle that very often the expansions and developments arising from internal forces within mathematics are just what the outside world needs.

Probably the most typical example is the invention of complex numbers: they emerged from the desire to solve equations, and they have become the basis of powerful theories essential to modern physics and without which developments like electronics would not exist. A second example is the theory of matrices, which were developed in mathematics to help understand the geometry of space around us. (A matrix is a rectangular array of numbers.) Physicists noticed that some of Heisenberg's computations resembled matrix calculations, so they tried replacing the position and momentum variables with matrices. The resulting theory became so transparent that they were scared to test it in more complicated systems for fear that the tests would not support the model. But the tests *did* support the model, and thereafter the

see Feature *on Page 2*

Events Calendar

More information about the following events will appear at our website at `http://www.math.usf.edu` as the event approaches. So mark the event on your calendar, and plan on taking part.

Nov. 12, 1999: **Mathematics Field Day.** We will be having a Field Day on Friday, Nov. 12. We invite high schools in Hillsborough, Manatee, Pasco, Pinellas, and Sarasota counties to send math teachers and high abil4nes outTd(Pi)1-1(oh)1(e)0saa-1(sning)-33ninU29(

Feature

continued from Page 1

development of the theory of infinite dimensional matrices ("operators"), driven by the theory of "function spaces" in mathematics went hand in hand with the development of quantum physics.

There are many applications of pure mathematics to the outside world. Some well known examples are: mathematical logic in the development of computers; stochastic processes in stock market predictions; prime factorization/number theory in secret codes (both military and public use); and harmonic analysis in signal processing and picture compression/recognition.

Some Clouds

Recently we have encountered a similar coincidence in our research at USF, in "approximation theory". (In our department there is a strong group working in approximation theory and related areas.) The main goal of *approximation theory* is to approximate complicated objects (like functions, surfaces, sets, etc.) by simpler ones (like polynomials, polygonal surfaces, ory of these more difficult weight functions are among our faculty: E. A. Rahmanov, E. B. Saff and recently M. H. Ismail. The major insight came from applying some purely mathematical methods with a root in physics: potentials in the presence of an "external field".

Approximation of Clouds

Over the last decade or so it has turned out that this more difficult theory has numerous applications in a variety of mathematical problems. E. B. Saff and I have been preparing a monograph on the subject (*Logarithmic Potentials with External Fields*) that was to be published in Springer's *Grundlehren der mathematischen Wissenschaften* series, which is probably the most prestigious old series of mathematical texts. One chapter of the book was devoted to the aforementioned orthogonal polynomials. Just before completing the manuscript, we learned that some theoretical physicists had been dealing with the same problem: describing the behavior of these more difficult orthogonal polynomials. They were working on statistical-mechanical models of quantum systems. They replaced quantum particles with mechanical balls, but to account for the quantum behavior they used a random distribution W of these balls and W could be a weight. Then orthogonal polynomials with respect to W emerge in the calculation of physical quantities like the density of states.

In science one often encounters "similar" questions, that, under closer scrutiny, turn out to be totally unrelated to what one is doing. However, our case was a perfect match, but the method of the physicists was completely different: they were using random matrix theory. We have been quite surprised by this unexpected coincidence, for our investigations were motivated by purely theoretical questions in approximation theory with no reference to physical applications whatsoever. As a result, we have become richer by an application of our methods, and at the same time provided a rigorous framework for proving results concerning random matrix models.

So after all is all mathematics relevant to the real world or should it be? This may be a question we should leave for philosophers to answer; but it is also a question that is debated in various forms ("good versus bad" mathematics; "pure versus applied" mathematics) over and over again. The anwer is probably "no", but in any case one should exercise some caution in harsh judgements: remember that one of the pearls of pure mathematics, the "little" Fermat theorem, had been sitting there as a beautiful but useless porcelain for over three hundred years before, in the late 1970's, it became the basis of public key cryptography and today the security of millions of bank transactions depend on it.

Let us finish with a quote from Eugene P. Wigner, who was a Nobel laureate physicist and at the same time an outstanding mathematician. "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research, and it will extend ... to wide branches of learning."

Events

continued from Page 1

Mar. 9, 2000: **Nagle Lecture.** The Nagle Lecture Series has invited Jerrold Marsden to give the

Faculty News

Last Spring, *Mourad Ismail* spent a semester at the Mathematical Science Research Institute at UC Berkeley after being invited to their half-year program on Random Matrics. He was also awarded